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Type-1

- Q 1. If $\sin A = \frac{3}{5}$, $0 < A < \frac{\pi}{2}$ and $\cos B = \frac{-12}{13}$, $\pi < B < \frac{3\pi}{2}$, find the following :
- (i) $\sin(A - B)$ (ii) $\cos(A + B)$ (iii) $\tan(A - B)$
- Q 2. If $\cot \alpha = \frac{1}{2}$, $\sec \beta = \frac{-5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$. Find the value of $\tan(\alpha + \beta)$. State the quadrant in which $\alpha + \beta$ terminates.
- Q 3. Find the values of the following :
- (i) $\tan 15^\circ$ (ii) $\tan 75^\circ$ (iii) $\tan 105^\circ$ (iv) $\tan \frac{13\pi}{12}$
- Q 4. Evaluate the following :
- (i) $\sin \frac{7\pi}{12} \cos \frac{\pi}{4} - \cos \frac{7\pi}{12} \sin \frac{\pi}{4}$ (ii) $\sin \frac{\pi}{4} \cos \frac{\pi}{12} + \cos \frac{\pi}{4} \sin \frac{\pi}{12}$ (iii) $\cos \frac{2\pi}{3} \cos \frac{\pi}{4} - \sin \frac{2\pi}{3} \sin \frac{\pi}{4}$
- Q 5. Prove that : $\cos\left(\frac{\pi}{4} - A\right) \cos\left(\frac{\pi}{4} - B\right) - \sin\left(\frac{\pi}{4} - A\right) \sin\left(\frac{\pi}{4} - B\right) = \sin(A + B)$
- Q 6. Prove that : $\sin(n + 1)A \sin(n + 2)A + \cos(n + 1)A \cos(n + 2)A = \cos A$
- Q 7. If $A + B = \frac{\pi}{4}$, prove that :
- (i) $(1 + \tan A)(1 + \tan B) = 2$ (ii) $(\cot A - 1)(\cot B - 1) = 2$
- Q 8. Prove that : $\frac{\tan(A + B)}{\cot(A - B)} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$
- Q 9. Prove that : $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$
- Q 10. Prove that : $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$
- Q 11. Prove that : $\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) = \frac{1}{\sqrt{2}} \sin A$
- Q 12. Evaluate the following :
- (i) $\sin 78^\circ \cos 18^\circ - \cos 78^\circ \sin 18^\circ$ (ii) $\cos 47^\circ \cos 13^\circ - \sin 47^\circ \sin 13^\circ$
 (iii) $\sin 36^\circ \cos 9^\circ + \cos 36^\circ \sin 9^\circ$ (iv) $\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ$
- Q 13. Prove that : (i) $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \tan 54^\circ$ (ii) $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$
 (iii) $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$
- Q 15. Prove that : $\sin(60^\circ - \theta) \cos(30^\circ + \theta) + \cos(60^\circ - \theta) \sin(30^\circ + \theta) = 1$

Answers

1. (i) $\frac{-16}{65}$ (ii) $\frac{-33}{65}$ (iii) $\frac{16}{63}$ 2. $\frac{2}{11}$ 3. (i) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (ii) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (iii) $-\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (iv) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
4. (i) $\frac{\sqrt{3}}{2}$ (ii) $\frac{\sqrt{3}}{2}$ (iii) $-\frac{\sqrt{3}+1}{2\sqrt{2}}$ 12. (i) $\frac{\sqrt{3}}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{\sqrt{2}}$ (iv) $\frac{1}{2}$

Type-2

Q 1. Prove that :

- (i) $\tan 8\theta - \tan 6\theta - \tan 2\theta = \tan 8\theta \tan 6\theta \tan 2\theta$ (ii) $\tan 15^\circ + \tan 30^\circ + \tan 15^\circ \tan 30^\circ = 1$
 (iii) $\tan 36^\circ + \tan 9^\circ + \tan 36^\circ \tan 9^\circ = 1$ (iv) $\tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \tan 9\theta \tan 4\theta$

Q 2. If $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$, then prove that $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = 1$

Q 3. If $\sin \alpha + \sin \beta = a$ and $\cos \alpha + \cos \beta = b$, show that

- (i) $\sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$ (ii) $\cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$

Type-3

Q 1. Find the maximum and minimum values of each of the following trigonometrical expressions :

- (i) $12 \sin \theta - 5 \cos \theta$ (ii) $12 \cos \theta + 5 \sin \theta + 4$
 (iii) $5 \cos \theta + 3 \sin\left(\frac{\pi}{6} - \theta\right) + 4$ (iv) $\sin \theta - \cos \theta + 1$

Q 2. Reduce each of the following expressions to the sine and cosine of a single expression :

- (i) $\sqrt{3} \sin \theta - \cos \theta$ (ii) $\cos \theta - \sin \theta$ (iii) $24 \cos \theta + 7 \sin \theta$

Q 3. Prove that $(2\sqrt{3} + 3) \sin \theta + 2\sqrt{3} \cos \theta$ lies between $-(2\sqrt{3} + \sqrt{15})$ and $(2\sqrt{3} + \sqrt{15})$

Answers

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|----|---------------------|----------------|---|
| 1. | minimum | Maximum | 2. (i) $2 \sin\left(\theta - \frac{\pi}{6}\right), -2 \cos\left(\frac{\pi}{3} + \theta\right)$
(ii) $\sqrt{2} \sin\left(\frac{\pi}{4} - \theta\right), \sqrt{2} \cos\left(\frac{\pi}{4} + \theta\right)$
(iii) $25 \sin(\alpha + \theta)$, where $\tan \alpha = \frac{24}{7}$,
$25 \cos(\theta - \alpha)$, where $\tan \alpha = \frac{7}{24}$ |
| | (i) -13 | 13 | |
| | (ii) -9 | 17 | |
| | (iii) -3 | 11 | |
| | (iv) $1 - \sqrt{2}$ | $1 + \sqrt{2}$ | |

Type-4

Q 1. Convert each of the following products into the sum or difference of sines and cosines :

(i) $2 \sin 5\theta \cos \theta$ (ii) $2 \cos 4\theta \cos 3\theta$ (iii) $2 \sin 3 \sin \theta$

(iv) $\sin 75^\circ \cos 15^\circ$ (v) $\cos 75^\circ \cos 15^\circ$

Q 2. Prove that : $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

Q 3. Prove that : $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$.

Q 4. Prove that : $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$

Q 5. Prove that :

(i) $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$

(ii) $\cos 40^\circ \cos 80^\circ \cos 160^\circ = -\frac{1}{8}$

(iii) $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$

(iv) $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$

(v) $\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = 3$

(vi) $\tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 80^\circ = 1$

(vii) $\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$

(viii) $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$

Q 6. Prove that $\sin A \sin (60^\circ - A) \sin (60^\circ + A) = \frac{1}{4} \sin 3A$

Q 7. Prove that $4 \sin \theta \sin \left(\frac{\pi}{3} + \theta \right) \sin \left(\frac{2\pi}{3} + \theta \right) = \sin 3\theta$

Q 8. Prove that : $4 \cos \theta \cos \left(\frac{\pi}{3} + \theta \right) \cos \left(\frac{\pi}{3} - \theta \right) = \cos 3\theta$

Q 9. Prove that : $\tan \theta \tan(60^\circ - \theta) \tan (60^\circ + \theta) = \tan 3\theta$

Answers

1. (i) $\sin 6\theta + \sin 4\theta$

(ii) $\cos 7\theta + \cos \theta$

(iii) $\cos 2\theta - \cos 4\theta$

(iv) $\frac{1}{2}[\sin 90^\circ + \sin 60^\circ]$

(v) $\frac{1}{2}[\cos 90^\circ + \cos 60^\circ]$

Type-5

Q 1. Prove that :

(i) $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$

(ii) $\frac{\sin 2\theta}{1 - \cos 2\theta} = \cot \theta$

(iii) $\frac{1 + \sin 2\theta + \cos 2\theta}{1 + \sin 2\theta - \cos 2\theta} = \cot \theta$

(iv) $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \frac{\theta}{2}$

(v) $\frac{\cos 2\theta}{1 + \sin 2\theta} = \tan \left(\frac{\pi}{4} - \theta \right)$

(vi) $\frac{\cos \theta}{1 + \sin \theta} = \tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right)$

Q 2. Show that : $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8\theta}}} = 2 \cos \theta$

Q 3. Prove that : $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

Q 4. Prove that :

$$(i) \cos^2 A + \cos^2 \left(A + \frac{2\pi}{3} \right) + \cos^2 \left(A - \frac{2\pi}{3} \right) = \frac{3}{2}$$

$$(ii) \cos^2 A + \cos^2 \left(A + \frac{\pi}{3} \right) + \cos^2 \left(A - \frac{\pi}{3} \right) = \frac{3}{2}$$

Q 5. Prove that : $\left(1 + \cos \frac{\pi}{8} \right) \left(1 + \cos \frac{3\pi}{8} \right) \left(1 + \cos \frac{5\pi}{8} \right) \left(1 + \cos \frac{7\pi}{8} \right) = \frac{1}{8}$

Q 6. Prove that : $\cos A \cos 2A \cos 2^2 A \cos 2^3 A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$

Q 7. Prove that : $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = -\frac{1}{8}$

Q 8. Prove that : $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{14\pi}{15} = \frac{1}{16}$

Q 9. Prove that : $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

Q 10. Prove that : $\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{7\pi}{14} \sin \frac{9\pi}{14} \sin \frac{11\pi}{14} \sin \frac{13\pi}{14} = \frac{1}{64}$

Q 11. Prove that : $\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{128}$

Q 12. If $\theta = \frac{\pi}{2^n + 1}$, prove that $2^n \cos \theta \cos 2\theta \cos 2^2 \theta \dots \cos 2^{n-1} \theta = 1$

Q 13. (i) If $\cos x = -\frac{3}{5}$ and x lies in the IIIrd quadrant, find the values of $\cos \frac{x}{2}, \sin \frac{x}{2}, \sin 2x$

(ii) If $\cos x = -\frac{3}{5}$ and x lies in IIInd quadrant, find the values of $\sin 2x$ and $\sin \frac{x}{2}$

Q 14. If $\sin x = \frac{\sqrt{5}}{3}$ and x lies in IIInd quadrant, find the values of $\cos \frac{x}{2}, \sin \frac{x}{2}$ and $\tan \frac{x}{2}$.

Q 15. If $0 \leq x \leq \pi$ and x lies in the IIInd quadrant such that $\sin x = \frac{1}{4}$. Find the values of

$$\cos \frac{x}{2}, \sin \frac{x}{2} \text{ and } \tan \frac{x}{2}.$$

Q 15. (i) If $\cos \theta = \frac{4}{5}$ and θ is acute, find $\tan 2\theta$

(ii) If $\sin \theta = \frac{4}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the value of $\sin 4\theta$

Q 16. Prove that : $\cos^3 A + \cos^3 (120^\circ + A) + \cos^3 (240^\circ + A) = \frac{3}{4} \cos 3A$

Q 17. Prove that :

$$(i) \tan A + \tan(60^\circ + A) - \tan(60^\circ - A) = 3 \tan 3A$$

$$(ii) \cot A + \cot(60^\circ + A) - \cot(60^\circ - A) = 3 \cot 3A$$

Q 8. $\tan A + \tan(60^\circ + A) - \tan(60^\circ - A) = 3 \tan 3A$

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Q 9. $\sin^3 A + \sin^3\left(\frac{2\pi}{3} + A\right) + \sin^3\left(\frac{4\pi}{3} + A\right) = -\frac{3}{4}\sin 3A$

Q 10. Prove that $|\sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta)| \leq \frac{1}{4}$ for all values of θ

Q 11. Prove that $|\cos \theta \cos(60^\circ - \theta) \cos(60^\circ + \theta)| \leq \frac{1}{4}$ for all values of θ

Q 12. If $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, then write the value of $\sqrt{\frac{1 + \cos 2\theta}{2}}$.

Q 13. If $\frac{\pi}{2} < \theta < \pi$, then write the value of $\sqrt{2 + \sqrt{2 + 2\cos 2\theta}}$ in the simplest form.

Q 14. If $\frac{\pi}{2} < \theta < \pi$, then write the value of $\sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}}$.

Q 15. If $\pi < \theta < \frac{3\pi}{2}$, then write the value of $\sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}}$.